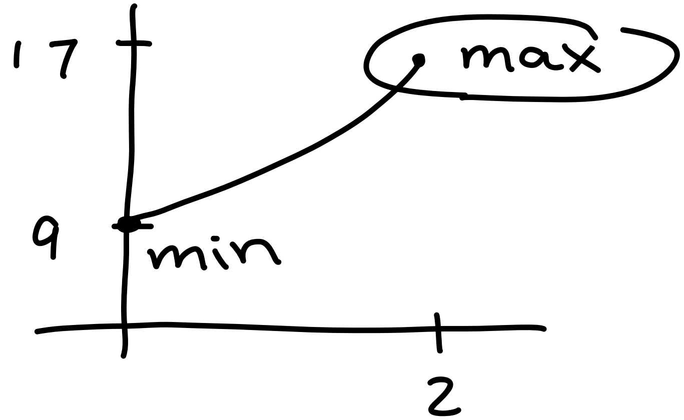
14.7 (part 2): Global Max/Min



2D Intro to Global Max/Min

First, a review of calculus 1…

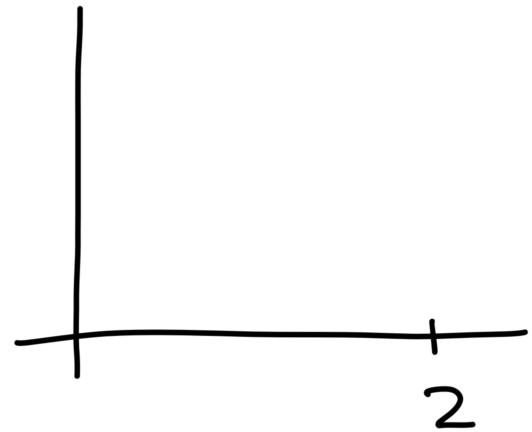
Do you remember how to do this?

Find the global max/min of:

A) $f(x) = 2x^2 + 9$ on $0 \leq x \leq 2$.

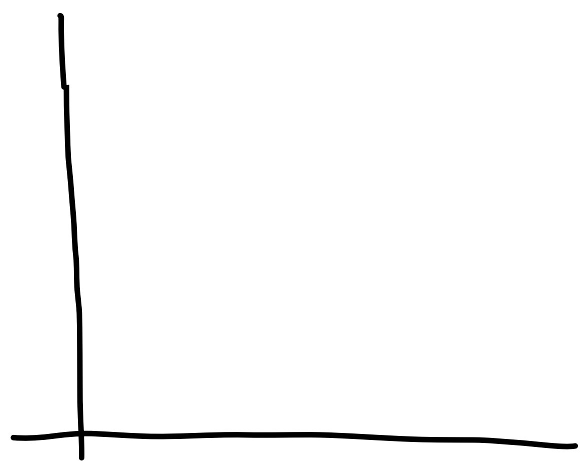
B) $g(x) = 4x^2 - 4x + 9$ on $0 \leq x \leq 2$.

C) $h(y) = 9$ on $0 \leq y \leq 4$.



must occur @
end point or
critical point

A $f'(x) = 4x = 0$ $x = 0$



Global (or Absolute) Max/Min:

<u>Given</u>

- A surface: $z = f(x, y)$
- A region on the xy-plane: R

<u>Want</u>

Biggest and smallest z over this region.

<u>How</u>

Step 1: Draw Region, label sides

Step 2: "Inside". Find all critical pts.

Step 3: "Boundary"

Over each boundary curve, plug into surface and solve the resulting one variable max/min problem.

Keep track of your outputs throughout.

biggest z = global max

smallest z = global min

F'18 – Exam 2 – Loveless

Find the global max of

$$f(x, y) = 4x^2 - xy + 9$$

over the triangular region with corners at (0,0), (0,4), and (2,4).

- *Solutions (see 3(b))*
- *Visuals: https://www.math3d.org/5lTfUKmA*

Sp'18 – Exam 2 – Loveless

Find the absolute max and min of $f(x,y) = 3y - xy$ over the region bounded by $y = x^2, y = 0, \& \ x = 4$.

- *Solutions (see 3(a))*
- **Visuals:** https://www.math3d.org/dsH5kXHT

W'15 – Exam 2 – Loveless

Find the global max and min of $f(x, y) = x^2 + 4y - x^2y + 1$ over the region bounded by $y = 0$ and $y = 2 - \frac{1}{2}x^2$.

- *Solutions (see 4(b))*
- **Visuals:** https://www.math3d.org/eaVxWB1i

15.1 Iterated Integrals – Double Integral Intro

Try this…

- Treat y as a constant.
- Integrate "inside" with respect to x.
- Then integrate with respect to y.

$$\int_0^2 \left(\int_1^2 8xy^2 \, dx \right) dy =$$

Try again in the reversed order:

$$\int_1^2 \left(\int_0^2 8xy^2 \, dy \right) dx =$$

Visuals: https://www.math3d.org/XZgRT8IT

15.1-15.2 Theory Overview

Given

1. $z = f(x, y)$
2. A region, *R*, in the *xy*-plane.

$$\iint_R f(x,y)dA = \lim_{m,n\to\infty} \sum_{i=1}^{m} \sum_{j=1}^{n} f(x_{ij}, y_{ij})\Delta A$$

= `signed’ volume between *f(x,y)* and *R*.

General Notes:

Gives a number!

- If *f(x,y)* is above the *xy*-plane it is *positive*.
- If *f(x,y)* is below the *xy*-plane it is *negative*.

Symbol Notes:

ΔA = area of base = $\Delta x \Delta y = \Delta y \Delta x$

$f(x_{ij}, y_{ij})\Delta A$ = (height)(area of base)

= volume of one approximating box

Units of $\iint_R f(x,y)dA$ are

(units of f(x,y))(units of x)(units of y)

Theory/Approximation Example

Example: Estimate the volume under

$z = f(x,y) = x + 2y^2$

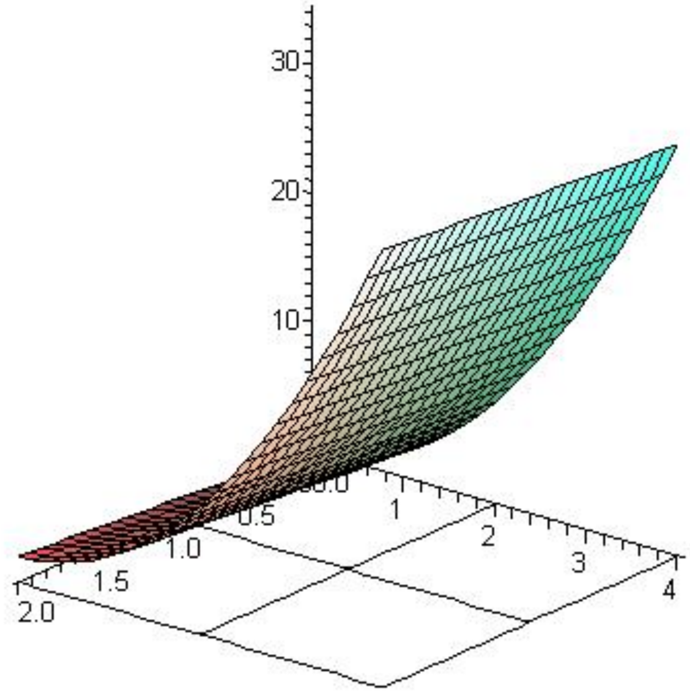
and above

R = [0,2] x [0,4]

= {(x,y) : 0 ≤ x ≤ 2, 0 ≤ y ≤ 4 }

(a) Break the region R into m = 2 columns and n = 2 rows; 4 sub-regions;

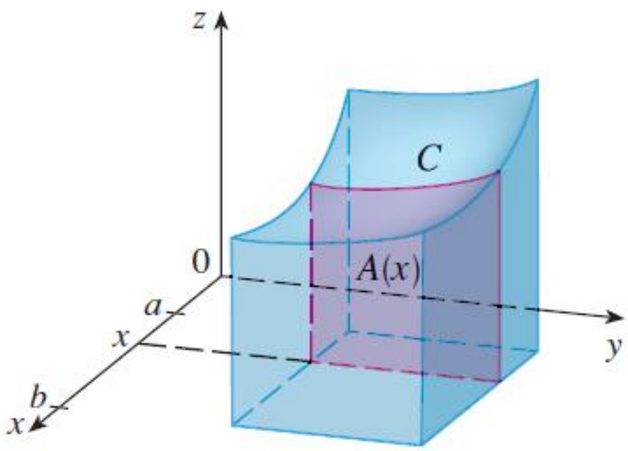
(b) Approx. using a rectangular box over each region (use *upper-right* endpts).



Iterated Integrals Theory

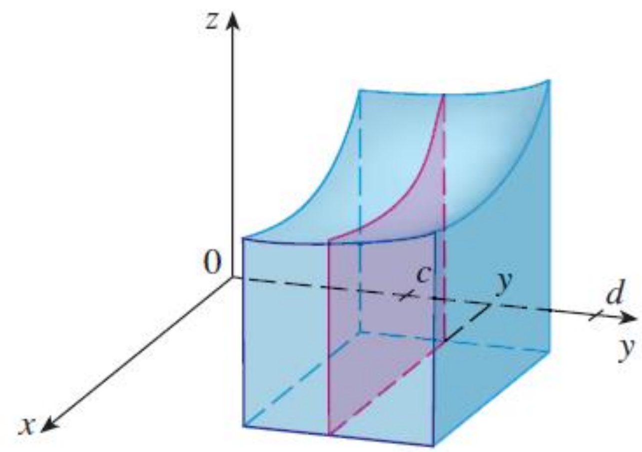
If you fix x: The area under this curve is

$$\int_c^d f(x,y)\,dy = \text{"cross sectional area under the surface at this fixed } x \text{ value"}$$



If you fix y: The area under this curve

$$\int_a^b f(x,y)\,dx = \text{"cross sectional area under the surface at this fixed } y \text{ value"}$$



From Math 125,

$$\text{Vol} = \int_a^b \text{Area(x)}dx = \int_a^b \left(\int_c^d f(x,y)dy \right) dx$$

$$\text{Vol} = \int_c^d \text{Area}(y)dy = \int_c^d \left(\int_a^b f(x,y)dx \right) dy$$

Two basic examples to help you visualize:

Visuals for (a): https://www.math3d.org/4HzHYfIJ

Visuals for (b): https://www.math3d.org/FL8kzBB1

You try to evaluate

$$(a) \int_0^3 \left(\int_0^2 5\, dx \right) dy$$

$$(b) \int_0^3 \left(\int_0^2 2 - x\, dx \right) dy$$

“Hard” 15.1 HW Problem

Compute

$$\iint_R \frac{2x}{1+xy}\,dA$$

over $R = [0,2]\ x\ [0,1]$.

Visuals: https://www.math3d.org/Ig0u6x7d

Preview of 15.2

Evaluate

$$\int_0^7 \int_{y/7}^1 2e^x \, dx \, dy$$

Also try evaluating (this should come out to the same number, see if it does)

$$\int_0^1 \int_0^{7x} 2e^x \, dy \, dx$$

Visual: https://www.math3d.org/YKdmMST9

More 15.2 Preview:

Calculus II review and 15.2 Warm up

Draw and describe the region bounded by

$$y = x^2, y = 2x + 3$$

(a) ...in terms of x?

(b) ...in terms of y?